2D21 Level-1 Compact Model v0.1

Scope

This document specifies the **Level-1 static compact model** developed for the General Electric 2D21 thyratron grid-current characteristics during anode conduction.

The modeled quantity is:

$$I_g = F(V_g, I_a)$$

where:

- V_g is control-grid voltage;
- I_g is control-grid current;
- I_a is anode current identifying the characteristic family.

The model is **family-parameterized**: for each characterized value of I_a, the parameters V_0, S_-, S_+, Q_A, and U_{20} are supplied.

Version 0.1 does **not** define a validated continuous law that predicts all of those parameters from arbitrary I_a.

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1. Normalized current

$$Y = \frac{I_g}{I_a}$$

The zero-crossing voltage V_0 is defined by:

$$I_g(V_0) = 0$$

Two normalized voltage coordinates are used:

$$X_- = \frac{V_g - V_0}{S_-}$$

for the negative branch, and

$$X_+ = \frac{V_g - V_0}{S_+}$$

for the positive branch.

The scale conventions are:

$$G_-(-1) = -0.010$$

and

$$G_+(1) = +0.005.$$

2. Positive branch

For $V_g \geq V_0$:

$$G_+(X_+; K) = 0.005\,\frac{\exp(KX_+) - 1}{\exp(K) - 1}$$

with the 2D21 device parameter:

$$K = 0.551.$$

Therefore:

$$I_{g,+} = I_a\,0.005\,\frac{\exp\left[K\frac{V_g - V_0}{S_+}\right] - 1}{\exp(K) - 1}.$$

The positive branch satisfies:

$$G_+(0) = 0$$

and

$$G_+(1) = 0.005.$$

3. Negative branch

For $V_g \leq V_0$:

$$G_-(X_-; Q_A, U_{20}) = -Q_A + \sum_{j=1}^{3} A_j e^{\lambda_j X_-}.$$

The frozen exponential constants are:

$$(\lambda_1, \lambda_2, \lambda_3) = (0.060,\ 0.240,\ 0.960).$$

The coefficients A_1, A_2, A_3 are **not free fit parameters**. They are determined from Q_A and U_{20} by imposing:

$$G_-(0) = 0,$$

$$G_-(-1) = -0.010,$$

$$G_-(-U_{20}) = -0.020.$$

Thus:

$$\begin{bmatrix} 1 & 1 & 1 \\ e^{-\lambda_1} & e^{-\lambda_2} & e^{-\lambda_3} \\ e^{-\lambda_1 U_{20}} & e^{-\lambda_2 U_{20}} & e^{-\lambda_3 U_{20}} \end{bmatrix} \begin{bmatrix} A_1 \\ A_2 \\ A_3 \end{bmatrix} = \begin{bmatrix} Q_A \\ Q_A - 0.010 \\ Q_A - 0.020 \end{bmatrix}.$$

The negative-branch current is:

$$I_{g,-} = I_a \left[-Q_A + \sum_{j=1}^{3} A_j \exp\left(\lambda_j \frac{V_g - V_0}{S_-} \right) \right].$$

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4. Complete Level-1 model

$$I_g(V_g, I_a) = \begin{cases} I_a \, G_-\left(\dfrac{V_g - V_0}{S_-}; Q_A, U_{20} \right), & V_g \le V_0, \\ I_a \, G_+\left(\dfrac{V_g - V_0}{S_+}; K \right), & V_g \ge V_0. \end{cases}$$

The model is continuous at the zero crossing:

$$G_-(0) = G_+(0) = 0.$$

Derivative continuity is **not imposed**. The model is therefore C^0, but not necessarily C^1, at V_0.

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5. Identified 2D21 family parameters

I_a (mA)	V_0 (V)	S_- (V)	S_+ (V)	Q_A	U_{20}
25	-0.28418	0.14187	0.03058	0.03136	4.995
50	-0.36110	0.23812	0.04413	0.02802	3.746
100	-0.43804	0.25758	0.06747	0.02819	4.984
150	-0.45898	0.32066	0.05269	0.02884	4.072
200	-0.50980	0.21483	0.05017	0.03254	4.592
300	-0.57868	0.07533	~0.014*	0.04810†	4.473

* S_+ at 300 mA is inferred from the model because the digitized curve does not reach the normalization level $Y = +0.005$.

† Q_A at 300 mA is weakly identified because the available negative branch does not fully expose the asymptotic plateau.

The 5 mA family is not included in the primary parameter table. Using the same negative-branch structure gives approximately:

$$Q_A \approx 0.044, \qquad U_{20} \approx 3.07.$$

It is treated as a possible low-current regime.

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6. Validation summary

For the negative branch, the Level-1 structure gives approximately:

I_a (mA)	RMSE (mA)	RMS residual / σ_{total}
25	0.00643	0.53
50	0.01082	0.93
100	0.02392	1.98
150	0.02601	1.83
200	0.01570	0.86
300	0.10485	1.24

Aggregate normalized RMS is approximately:

$$1.33\,\sigma.$$

Downsampling the numerical representation from roughly 300 to roughly 30 points per family changes Q_A by less than about 0.2% and U_{20} by less than about 0.3% for the 25–200 mA families.

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7. Parameter hierarchy

Device/global structure

- $K = 0.551$ for the current 2D21 Level-1 model.
- $(\lambda_1, \lambda_2, \lambda_3) = (0.060, 0.240, 0.960)$.
- Positive and negative functional forms are frozen in v0.1.

Family-specific parameters

For each characterized I_a:

- V_0
- S_-
- S_+
- Q_A
- U_{20}

Version 0.1 does not claim validated continuous laws:

$$V_0(I_a),\ S_-(I_a),\ S_+(I_a),\ Q_A(I_a),\ U_{20}(I_a)$$

sufficient to generate arbitrary intermediate-current families.

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8. Limitations

This Level-1 model is static and does not include:

- trigger dynamics;
- ionization time;
- deionization and recovery;
- latch behavior;
- interelectrode capacitances;
- full G_2 dependence;
- complete conducting-state behavior;
- temperature, gas-pressure, aging, or history dependence.

It should therefore be treated as a static grid-current submodel, not as a complete SPICE model of the 2D21.

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9. Version

Model: **2D21 Level-1 Compact Model**
Version: **0.1**
Status: mathematical structure frozen for the current 2D21 study.